Toward a Unified Theory of Proportion

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Introduction

Proportional reasoning is a ubiquitous part of the human experience. We engage in proportional reasoning to meet both informal and specialized goals across a range of domains, such as medicine (e.g., disease rates, drug dosages), finance and commerce (e.g., interest rates, discounts), cooking and baking (e.g., scaling ingredient amounts), and many others. Given this variation in usage, it may not be surprising that proportional reasoning does not have a singular definition or interpretation, but instead is a complex topic with many interconnected concepts. The central goal of this symposium is to shed light on this complexity by discussing diverse perspectives of proportional reasoning.

Proportion Tracking as a Learning Mechanism

Substantial work suggests that human infants are able to track proportional information (McCrink & Wynn, 2006; Duffy, Huttenlocher, & Levine, 2005) and use it to make probabilistic inferences (Denison, Reed, & Xu, 2013), inferences about populations from samples (Xu & Garcia, 2008), and inferences about other people's preferences (Kushnir, Xu, & Wellman, 2010). Furthermore, these early abilities to use proportional information may form the basis of a powerful domain general learning mechanism early in development (Xu & Kushnir, 2013).

Speaker: Stephanie Denison

Denison will discuss the current state of evidence on the earliest developments of proportion-based probabilistic reasoning. Although substantial research suggests that infants, non-human primates, and other species can engage in proportional and probabilistic reasoning (e.g., Denison et al., 2013; Eckert et al., 2018; Teglas et al., 2007), there are mixed results in 3- and 4-year-olds' abilities to make similar inferences (Girotto et al., 2016; Gualtieri & Denison, 2019). This talk will raise a number of possible causes of these age differences, including whether current methods mask the abilities of preschoolers or whether infant methods should be revisited. She also examines how differences in conceptual development (i.e., the maturity of preschoolers versus infants and non-human animals) may contribute to preschoolers' apparent dip in performance.

Proportion as One Source of Information

Additionally, by 6-years-old children begin to show systematic errors in proportional reasoning in certain contexts. When the stimuli involve countable information (e.g., the number of pieces), children may erroneously decide that 2/3 < 3/7 because 2 < 3 (Boyer, Levine, & Huttenlocher, 2008). Importantly, children are able to reason proportionally when the stimuli are not divided (i.e., countable number is not available; Boyer et al., 2008). Together, these findings suggest that children's difficulties with proportional reasoning at this age may be due to a strategy change (i.e., over-use of counting) in contexts that impact the saliency of different information (i.e., number versus proportion).

Speaker: Michelle Hurst

Hurst will discuss her work investigating what impacts young children's tendency to use proportional versus numerical quantities to make judgements across a range of domains, including probability (Hurst & Cordes, 2018), category learning (Hurst & Cordes, 2019), social evaluations based on resource distribution, and children's interpretation of the word "most". Although there are domain-specific strategies, the data suggest that some perceptual features impact proportional reasoning similarly across a range of domains.

Ratio as a Type of Quantity

Akin to absolute magnitude, which can be extracted from discrete sets (i.e., the number of items) or continuous formats (e.g., length, area), proportional magnitude can be processed via the ratio of discrete number, 1D extent, 2D area, and so on. Although there is substantial work investigating the developmental and psychophysical characteristics of absolute magnitude representations (Brannon, Lutz, & Cordes, 2006; Halberda & Feigenson, 2008; Leibovich & Henik, 2013), researchers have only recently begun to investigate proportion magnitude in this way.

Speaker: Yunji Park

Park will discuss her work investigating people's ability to track both absolute and proportional quantity across development (preschools, 2nd grade, 5th grade, and adults) and format (dots, lines, circles, and irregular blobs). Overall, performance does vary across format type and development and the pattern of format differences was the same for both absolute and ratio comparisons. However, performance on a given ratio comparison format was more related to performance on the other types of ratio than the absolute comparison in the same format. These results highlight both similarities and differences in how absolute and relative quantities are represented across formats and development.

Ratio as an Abstraction

Decades of psychophysical research has shown that quantitative representations are encoded in an analog format wherein discriminations among values are limited by the ratio between them (e.g., Moyer & Landauer, 1967). The mental scaling of quantities is thus inherently proportional.

Speaker: Jessica Cantlon

Cantlon will discuss data showing that adults and children compare different continua spontaneously by translating them into a common psychological space that preserves their ratio relations. Adults spontaneously track similarities between sequences of height, size, loudness, brightness, and pitch based on relative, proportional changes in values for each dimension. This proportional representation emerges by at least 5 years of age – when children compare sequences of different lengths, they align the sequence endpoints and compare their relative values. A computational function of proportional scaling is that it is useful (if not necessary) for comparing quantities between modalities and dimensions.

Discussion

Overall, Denison's expertise in infant proportional reasoning and cognition, Hurst's expertise in proportional reasoning across development and contexts, Park's expertise in ratio magnitude representation across formats, and Cantlon's expertise in magnitude representation will allow for broad and deep discussion of the open questions and theoretical implications of these distinct perspectives. The discussion will focus on how these varying perspectives inform each other, where they diverge and provide different predictions or implications, and what important next steps are necessary for developing and testing a unified theory of proportional reasoning. By bringing together scholars investigating proportional reasoning from varying perspectives, this symposium aims to bridge across these disparate research silos. Our hope is that new insights will emerge and collaborations will be formed that will allow for substantial and continued progress in understanding how the human mind represents proportional information.

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