Is probability utility correlation really correlation? An individual-level analysis of risk-reward heuristics

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Abstract

Utility and probability have been considered independent constructs for decision making under uncertainty. However, many studies have suggested that people assume there is a correlation between probability and utility. Some studies have demonstrated that people appear to estimate the utility of events depending on their probabilities, and other studies recently indicated the existence of "risk-reward heuristics" that assume a negative correlation between probability and utility in the real world when inferring winning probabilities from payoffs during decisions made under uncertainty. This study aimed to explore the relationship between probability and utility by requiring participants to estimate both probabilities from payoffs and payoffs from probabilities under a gain or loss situation. The results indicated that when estimating values of payoffs from probabilities, participants' judgments showed clear negative correlations between probability and utility both in the gain and loss condition. However, when estimating probabilities from payoffs, this negative correlation between utility and probability was found only in a gain situation. These results support the existence of risk-reward heuristics, and at the same time, suggest a possibility that people have different intuitions for the probability-utility relationship between the gain and loss domains.

Keywords: probability. utility, risk-reward heuristics, individual-level analysis, loss domain

Background

The most fundamental issue in decision theory is how to deal with utility and probability. Existing decision theories such as expected value theory (Pascal, 1665), expected utility theory (Von Neumann & Morgenstern, 1944), or prospect theory (Kahneman & Tversky, 1979) have aimed to represent the unknown future in terms of uncertainty and desirability of events, and construct quantity that can guide decisions by integrating factors like the uncertainty and desirability of the decision. Since Von Neumann and Morgenstern (1944) have constructed its theoretical formalization, expected utility theory has been considered a normative framework to express decision making under uncertainty. Expected utility is defined as the linear sum of utilities for the possible outcomes weighted by their probabilities,

$$E[u(x)] = \sum_{i=1}^{n} p_i u(x_i), \qquad (1)$$

where x_i indicates a possible outcome, p_i indicates the probability for an outcome, x_i, and $u(x_i)$ indicates the utility function for x_i.

In this theory, the desirability of an outcome is assumed to be reflected as utility, and the uncertainty of an outcome is assumed to be reflected as a probability. In addition, expected utility theory also assumes that probability and utility are considered independent constructs: utility is assumed to reflect only the outcome desirability of an event, not the uncertainty; and probability is assumed to reflect only the uncertainty, not the outcome desirability of an event. However, many studies on judgment and decision making have demonstrated a dependency on probability judgments for future outcomes on their magnitudes or utilities (e.g., Hahn & Harris, 2014; Harris, Corner, & Hahn, 2009; Krizan & Windschitl, 2007). This proposition was suggested by early research on decision making (Crandall, Solomon, & Kellaway, 1955; Edwards, 1962; Marks, 1951; Morlock & Hertz, 1964), but controversy remains with regards to how valence of outcome/utility affects probability judgment (Edwards, 1962; Fischer & Jungermann, 1996; Harris & Corner, 2011; Massey, Simmons, & Armor, 2011; Simmons & Massey, 2012; Weber & Hilton, 1990). For example, while Fischer and Jungermann (1990) found that, as a whole, the individuals' estimated probabilities for verbal probability phrases were lower for negative outcomes compared to positive outcomes in their study, Weber and Hilton (1996) reported the opposite tendency. To settle this controversy, Harris et al. (2009) examined the effect of utility on probability under experimentally controlled conditions and found that the seriousness of an event increases the estimated probabilities for the outcomes. Although the mechanism and direction of the effect are still unclear, previous studies suggest a relationship between probability judgments for outcomes and their utilities.

A recent theoretical development about the correlation between probability and utility is the study by Pleskac and Hertwig (2014). From examinations of real-world data such as gambling behaviors, finance or publications of scientific papers, Pleskac and Hertwig (2014) found a correlation between outcome desirability and their and likelihoods: large payoffs for the gambles are associated with lower winning probabilities; and the acceptance rate for scientific journals decreases as their values of impact factor increase. From these real world examinations, they proposed that estimating the inverse proportionality between probability and outcome desirability is ecologically rational. In their study, participants were also required to estimate their winning probabilities for various gambles that had different payoff values. The results demonstrated a negative correlation between estimated probabilities and payoff values. From the results of both the real-world statistics and psychological experiments, Pleskac and Hertwig (2014) insisted that individuals' estimations of probabilities for outcomes from its utility are based on an ecologically rational strategy as it

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exploits the statistical structure of gambling environments to substitute the missing information of probability. Pleskac and Hertwig (2014) call this strategy risk-reward heuristics, and several behavioral experiments supported people's use of heuristics (Hoffart et al., 2019; Skylark & Prabhu-Naik, 2018).

Existing studies demonstrate that people see a relationship between probability and utility, and their perception of this relationship might be based on knowledge about real-world data. Considering these findings, this paper aimed to explore the following two problems that had previously not been paid any attention. These two problems concern how people consider the relationship between probability and utility, and sheds light on another aspect of the issue of probability-utility correlation.

Is the correlation between probability and utility really "correlational"?

The first purpose of this study is to explore how people consider the probability-utility correlation more profoundly in terms of the meaning of "correlation." A main assumption of existing studies is that people's perception between probability and utility is correlational (e.g., Edwards, 1962; Pleskac & Hertwig, 2014; Hoffart et al., 2019). Correlation is defined as a statistical association between two variables that is commonly expressed as the degree to which a pair of variables is linearly associated. Therefore, correlation refers to an asymmetrical relationship: the magnitude of utility would vary in accord with the value of probability, and vice versa. To determine the intercorrelation between probability and utility, existing studies performed experiments in which values of probabilities (e.g., Skylark & Prabhu-Naik, 2019) or utilities (Edwards, 1962; Hoffart et al., 2019; Pleskac & Hertwig, 2014) of outcomes were manipulated and the subsequent effects on utility or probability judgments were examined. For example, Hoffart et al. (2019) manipulated values of payoffs for gambles and examined whether participants' probability judgment would vary in accord with the manipulation of the payoff values. In the same way, Skylark and Prabhu-Naik (2019) examined the correlation between probability and utility by manipulating the values of probability of the gambles and analyzed participants' estimations for the payoff values. By these experimental procedures, existing studies have insisted that people assume a relationship between probability and utility.

However, these procedures can only demonstrate unidirectional effects of the independent variables on dependent variables separately and thus, cannot determine bidirectional relation between the variables. This is due to a possibility that participants who thought probability affects utility did *not* consider that utility affects probability, and vice versa. In other words, to demonstrate the correlation between probability and utility, it is necessary to show the effect of utility on probability judgment and that of probability on utility judgment in the *same* participants. In one's knowledge, no study has performed an empirical examination that satisfies this condition. Thus, strictly speaking, existing studies could not determine whether the relationship between utility and probability would be correlational or not. Therefore, the first purpose of this study is to explore risk-reward heuristic by individual-level analysis.

Does the probability-utility correlation exist in the loss domain?

The second purpose of this study is to examine the inverse proportionality in the loss domain. The existing studies (e.g., Hoffart et al., 2019; Pleskac & Hertwig, 2014) have mainly treated a gamble situation where participants were required to answer their winning probabilities or values of payoff that they could obtain when winning the gamble. Thus, these studies can be considered as paying attention only to the gain domain. An asymmetry between gain and loss is one main feature of human decision making (e.g., Kahneman & Tversky, 1979; Tversky & Kahneman, 1981). Thus, if and how probability-utility correlation would work in the loss domain serve as interesting questions to explore.

The exploration of the loss domain is also theoretically important for risk-reward heuristic approach (Pleskac & Hertwig, 2014). As stated above, the use of risk-reward heuristics is considered a result of reflecting regularities of the real world. In everyday life, the correlation between probability and utility can also be found in the loss domain. For example, Stewart, Brown, and Chater (2006) reported that debt frequency decreases as the amount of debt increases. The magnitudes of earthquakes are negatively correlated with their frequencies. The correlation between the acceptance rate of scientific articles and their impact factor reported in Pleskac and Hertwig (2014) can also be positioned as an example for loss domain when considering the acceptance rate as an inverse number of the rejection rate. Although not as prominent as the gain domain, the correlation between probability and utility can also be found in the loss domain in the real world. Thus, people may consider the correlation between probability and utility also holds in the loss domain.

Purposes of this study

This study aimed to explore probability-utility correlation by examining the bidirectional relationship between probability and utility, and by exploring the correlation in the loss domain. In doing so, this study required participants to estimate both the probability from utility and from utility and probability under gain or loss conditions. Study 1 compared probability-utility correlation between gain and loss conditions by between-participants design, and Study 2 performed this by within-participants design.

Study 1

Participants and procedure

A total of 93 undergraduates who were naïve to psychological experiment participated in Study 1. The presentations of stimulus and the measurement of dependent variables were performed using Google Forms. Participants estimated reward probabilities or values of payoffs for two-outcome gambles under gain or loss conditions. As a probability judgment task in the gain condition, for example, participants were required to answer the following question:

> Here is a gamble that you get 240 yen if you win. What probability do you think to win this gamble?

For the probability judgment task, the outcomes were zero or a specifically known reward (reward magnitudes [in Japanese yen]: 240, 400, 470, 1,200, 1,600, 2,800) as followed by Hoffart et al., (2019).

As utility judgment task in the loss condition, for example, participants were required to answer the following question;

> Here is a gamble that has 1% of losing money. What amount of money do you think you will lose when you lose this gamble?

For the utility judgment task, the probabilities were also specific values (1%, 17%, 33%, 50%, 67%, 83%, 99%).

Results and discussion

Following Skylark and Prabhu-Naik (2018), Study 1 applied a logarithmic transform ($\log(x+1)$) to the values of both the independent and dependent variables. Figure 1 demonstrates average participants judgments of probability and reward estimation tasks both in gain and loss conditions. As these graphs indicate, with regards to the results in gain condition, probabilities and reward magnitudes appear to be negatively correlated both in the probability and reward judgment tasks. Regression analysis demonstrated a significant negative relationship between the reward magnitudes and estimated probabilities (beta=-0.01, 95% confidence interval: -0.017 to -0.003) and a significant negative effect of probabilities on the outcome magnitude estimation task (beta=-0.02, 95% confidence interval: -0.027 to -0.013). Thus, these results support a bidirectional association between probability and rewards: probability values affected reward judgment, and reward magnitudes, in turn, affected probability judgments.

However, the results of the loss condition shown in Figure 1 and 2 also demonstrate that participant estimates did not obey the negative correlation between probability and rewards magnitude. The coefficient between probability and reward magnitude did not become significant in reward judgment task (beta=-0.009, 95% confidence interval: -0.015 to 0.005), and it did not demonstrate a significant effect of the rewards values on probability judgment the probability estimation tasks (beta=-0.001, 95% confidence interval: - 0.002 to 0.0001).

In addition to the above group-level analyses, Study 1 also performed the same correlational analyses to individual data and examined whether the judgment of individual participants obeyed the negative correlation between probability and utility. Figure 2 shows boxplots for the results of these analyses: values of the correlation coefficients between the probability and outcome magnitude were distributed below zero in the gain condition, but not in the loss condition. Whereas mean values of correlation coefficients between probability and outcome magnitude were below zero both in outcome magnitude (mean=-0.74, 95% confidence interval: -0.60 to -0.86) and probability judgment (mean=-0.60, 95% confidence interval: -0.43 to -0.76) in the gain condition, the mean value of the correlation coefficient between probability and outcome magnitude was not below zero in outcome magnitude task (mean=-0.27, 95% confidence interval: -0.07 to -0.46), and it was not so in probability judgment task (mean=-0.14, 95% confidence interval: -0.37 to -0.10) in the loss condition. These results correspond to those for group-level analyses, indicating that the existing findings (Hoffart et al., 2019; Pleskac & Hertwig, 2014; Skylark & Prabhu-Naik, 2019) were not an artifact due to averaging the individual data.

Another focus of this study was whether probability-outcome magnitude association is bidirectional or not. Exploration for this point requires an examination into whether correlations between probability and outcome co-occur in the same participants. Figure 3 demonstrates scatterplots for the distribution of correlation coefficients between probability and outcome magnitudes both in probability and reward magnitude judgment tasks. This graph shows that the association between probability and outcome is bidirectional in the gain condition, but not so in the loss condition. In the former condition, most of the participants showed negative correlations between probability and utility both in the probability judgment and reward judgment task, whereas in the latter condition, participants distributed uniformly over the four regions. Due to the sparseness of data in the first and fourth orthants, Study 1 could not perform a statistical test to examine the deviation from the uniform distribution. However, the distribution of the correlation coefficients suggests that the strength of association between probability and reward was different between gain and loss condition.

Thus, these results implicate the following two points. First, they demonstrate a bidirectional relationship between probability and utility in the same participants in the gain condition; and participants who considered that higher probabilities were associated with lower probabilities also

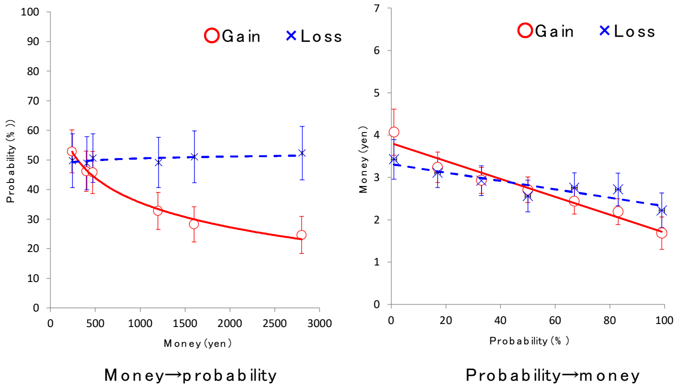

Figure 1. Average estimates of probability and utility judgments of Study 1. The left panel demonstrates results of probability judgement, and the right panel demonstrates the results of reward judgment.

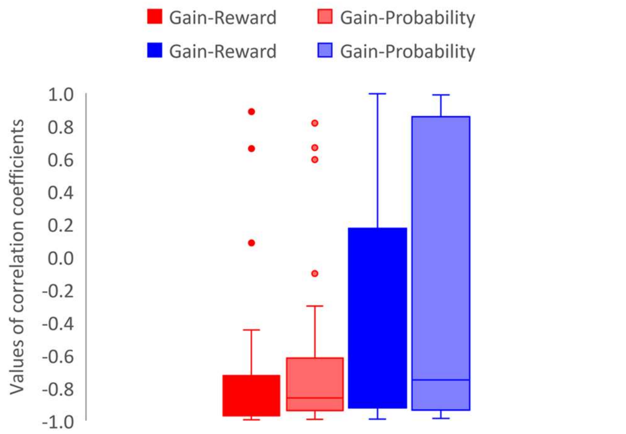

Figure 2. Boxplot of individual correlation coefficients between probability and outcome magnitude in Study 1.

considered that higher utilities were associated with lower probabilities. These results are replicated the existing findings that demonstrated the correlation between probabilities and utilities (Hoffart et al., 2019; Pleskac & Hertwig, 2014; Skylark & Prabhu-Naik, 2019), and are important in indicating that this correlation is correlational. As far as one knows, this is the first example that demonstrates the effect of probability on utility judgment and that of utility on probability judgment in the same participants.

Study 2

The main purpose of Study 2 was to replicate the findings in Study 1 under within-participants design. To accomplish this, Study 2 required participants to answer probability and outcome magnitude tasks both in the gain and loss conditions.

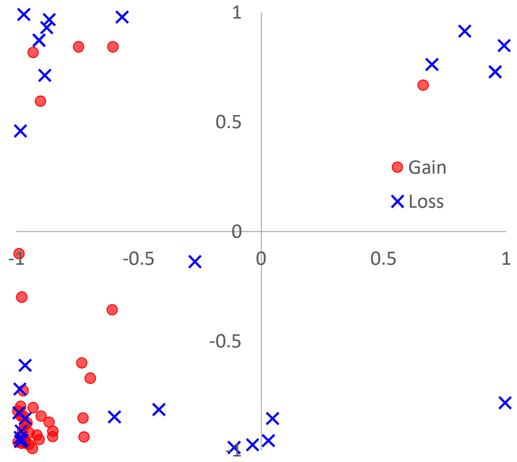

Figure 3. Scatterplots for distributions of participants on correlations between probabilities and utilities in Study 1. The horizontal axis indicates values of correlation coefficients between probability and payoff in reward magnitude judgments.

Participants and procedure

Participants in Study 2 comprised 70 undergraduates, and the study adopted almost the same procedure as in Study 1 except that Study 2 employed the within-participants design for the gain and loss condition; that is, the participants answered the probability and outcome magnitude judgment tasks both for gain and loss domains.

Second, Study 1 also demonstrates that association between probability and utility in the loss domain is not as strong as that in the gain condition. With regards to the probability judgment task, the correlation between probability and utility was not statistically significant at group-level analysis.

Additionally, contrary to the results in the gain condition, individual analyses also demonstrate large variations among participants in correlations between probabilities and rewards in the loss domain. These results suggest that the association between probability and outcome magnitude in loss domain might not be as strong as that in the gain domain, and it is not consistent throughout the participants and consistently indicates that participants; judgments for probability and utility are different between gain and loss.

Results and discussion

Following Skylark and Prabhu-Naik (2018), Study 2 also applied a logarithmic transform (log(x+1)) to participants' judgments for the magnitude of outcome. With regards to the gain condition, results shown in Figure 4 demonstrate the same trends as those in Study 1: probabilities and utilities appear to be negatively correlated in the gain condition. Regression analysis in the same way as Study 1 demonstrated a significant negative relationship between the reward magnitudes and estimated probabilities both in the probability judgment task (beta=-0.01, 95% confidence interval: -0.017 to -0.005), and the outcome magnitude estimation task (beta=-0.005, 95% confidence interval: -0.006 to -0.003). Thus, these results support a bidirectional association between probability and rewards: probability values affected reward judgment, and reward magnitude also affected probability judgments.

Study 2 also performed the same analyses to the data for loss condition. In contrast to Study 1, the regression analysis demonstrated a significant negative relationship between the reward magnitudes and estimated probabilities (beta=-0.005, 95% confidence interval: -6.01 to -3.75) in loss domain although its beta value was outside the 95% confidence interval of the beta value for the gain condition. The regression analysis for the magnitude estimation task demonstrated a significant effect of probability on reward magnitude (beta=-0.009, 95% confidence interval: -0.01 to -0.08).

Study 2 also performed individual-level analyses in the same way as Study 1. Figure 5 shows the boxplots for the correlation coefficients between probability and reward magnitude in the gain and loss conditions. This graph also demonstrates almost the same trends as that in Study 1: values of correlation coefficients between probability and outcome magnitude appear to be distributed below zero in the gain condition, but not in the loss condition. However, in Study 2, mean values of correlation coefficients between probability and outcome magnitude were below zero both in the outcome magnitude (mean: -0.71 and -0.32; 95% confidence interval: -0.76 to -0.67 and -0.49 to 0.16; gain condition and loss condition, respectively) and probability judgment (mean:-0.80 and 0.40; 95% confidence interval: -0.87 to -0.73 and -0.56 to -0.23; gain condition and loss condition respectively) both in the gain condition and loss condition, although ranges of the 95% confidence intervals did not overlap each other.

Figure 4 that shows the results for associations in the correlations indicates the same trend as that of Study 1. In gain condition, most of the correlations between probability and utility both become negative both in probability and reward judgment tasks, but not so in loss condition. Chi-square test that tried to examine the equality of distribution between gain and loss condition demonstrated a significant difference between the two conditions ($x^2(3) = 25.71, p < .001$).

General discussion

The results of the two studies can be summarized as follows:

First, this study confirmed the probability-utility correlation within the same participants. In one's knowledge,

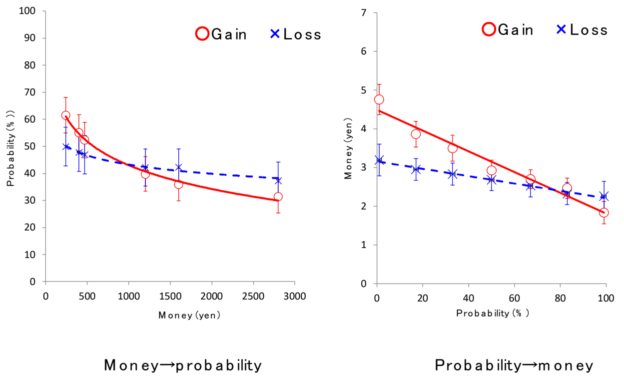

Figure 4. Average estimates of probability and utility judgments of Study 2: left panel demonstrates results of probability judgement, and right panel demonstrate the results of reward judgment.

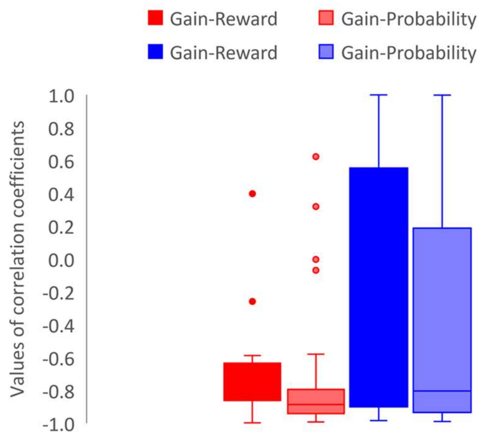

Figure 5. Boxplot of individual correlation coefficients between probability and outcome magnitude in Study 2.

this is the first example that demonstrated probability-utility correlation in the same participants. Although expected utility theory assumed independence between probability and utility (Edwards, 1962; Von Neumann & Morgenstern, 1944), many studies have suggested that people assume a correlation between probability and magnitude of outcome (Crandall, Solomon, & Kellaway, 1955; Edwards, 1953; Edwards, 1962; Irwin, 1953; Marks, 1951; Morlock & Hertz, 1964), and recent studies (Hoffart et al., 2019; Pleskac & Hertwig, 2014) have tried to show that people's perception of the relationship between probability and utility might be based on knowledge from the natural environment. This study pays attention to whether people's perception for the relationship correlational and demonstrated that in the gain condition, people consider that probability affect reward judgment and at the same time magnitude of rewards also affect probability judgment.

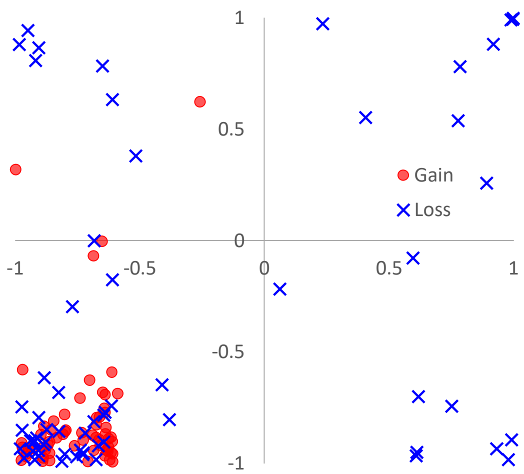

Figure 6. Scatterplots for distributions of participants on correlations between probabilities and utilities in Study 2. Meanings of the horizontal and vertical axis are the same as Study 1.

One more finding of this study is the difference in the perception of the correlation between probability and magnitude of outcome between the gain and loss condition. The asymmetry between gain and loss (Kahneman & Tversky, 1979; Tversky & Kahneman, 1981) is one of the most established findings in the domain of judgment and decision-making research. Although the existing studies have reported a difference in the effect on probability judgment between gain and loss (Edwards, 1962; Harris et al., 2009), the difference in how people associate probability and magnitude of the outcome had not been explored. This study can also be positioned as a first example which demonstrated that people associate the probability and magnitude of outcome differently between gain and loss. More precisely, this study demonstrates that association between probability and outcome might be weaker in the loss domain than that of the gain domain.

This finding is also important when considering the origin of the association between probability and magnitude of outcome. One main theoretical explanation for the association between probability and magnitude of the outcome is experience from the natural environment (Hoffart et al., 2019; Pleskac & Hertwig, 2014). Intuitively, although people know several examples such as depth (Stewart et al., 2006) or magnitude of an earthquake, the examples of association between probability and the magnitude of the outcome are not so prominent and available in the loss domain as those for the gain domain. All the examples reported in Pleskac and Hertwig (2014) are can be considered as those of gain domain. Considering this point, this study might provide supporting evidence for the position that probability-outcome association is based on learning from the natural environment because unavailability of the examples for loss domain might reflect lack of learning from the natural environment.

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