Practical Interpretation and Insights with Recurrence Quantification Analysis for Decision Making Research

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Keywords: decision making; choice behavior; recurrence quantification analysis; visual analytics; data sequences; discrete measurements

Introduction

A cornerstone of behavioral modeling and decision-making research includes characterizing the strategies people employ to make decisions. We often study strategies by collecting sequences of choices which hold considerable information about human cognition and behavior. However, it can be difficult to identify patterns and extract strategies, (e.g., 'win-stay-lose-shift'), from noisy, raw sequence data. Consequently, many standard analytical approaches aggregate choice data (e.g., over time, across multiple participants, etc.). Unfortunately, this process obscures temporal patterns that may provide insight about strategies employed, strategy switches, or even adaptation of strategies over time. As illustrated by McCormick, Blaha, and Gonzalez (2020b), we can generate novel insights about decision making strategies by characterizing patterns over choice sequences with recurrence quantification analysis.

Recurrence quantification analysis (RQA) is a visual analytic approach, comprised of a recurrence plot (RP) and set of related statistics, that allows researchers to visualize and quantify the dynamics of event states repeating or shifting over time. RQA offers precision for quantitatively characterizing sequences of observations while preserving temporal patterns in sequences. We can use RQA to better uncover cognitive strategies captured in choice sequence dynamics. RQA was originally developed to assess time series data and complex systems in Physics (Webber & Marwan, 2014). Both RPs and RQA statistics have been successfully adopted by cognitive scientists for both continuous and discrete data, with applications in interpersonal dynamics, conversation, motor control (Coco & Dale, 2014), and team communication (Gorman, Cooke, Amazeen, & Fouse, 2012).

Despite its promise, to successfully use RQA, researchers must first learn to interpret RPs and RQA statistics. We build on previous tutorials introducing RQA to cognitive scientists (Coco & Dale, 2015, 2014; Wallot & Leonardi, 2018) by providing a course on both how to implement discrete RQA on choice sequences and how to develop the visual expertise to quickly interpret RPs and derive insights about cognitive strategies in data. This tutorial offers researchers hands-on activities to develop (or improve) their ability to interpret RQA and apply it to their own data.

Sample RQA Strategy Characterization

This tutorial will focus on characterizing cognitive strategies in choice sequences, where *choice sequence* refers to discrete times series data with finite numbers of states or choice options on each trial. Below, we demonstrate several characterizations tutorial participants can learn to make.

Figure 1 depicts four simulated choice strategies for 50 consecutive trials of a two-alternative, forced-choice task. These four strategies can be verbalized as: a uniform probability of selecting Choice 1 or 2 (Uniform), an increasing probability of selecting Choice 2 (Increasing), an strong early preference for Choice 1 and a shifting later preference for Choice 2 (Exploit-Then-Explore), and an initially higher probability of selecting Choice 1 that shifts into a low probability of Choice 1 then ultimately returns to a higher probability of selecting Choice 1 (U-shaped). Increasing and Exploit-Then-Explore show similar proportions of Choice 2, but differences in individual propensities for exploring the choice options. While these four strategies are might be comparable on an aggregated metric of Choice 2 proportion (ranging from 44% to 58%), they can be visually distinguished in recurrence plots (and also by recurrence statistics such as average diagonal line length). When one can quickly interpret RPs, it can be far easier to visually inspect multiple choice sequences for such patterns. Additionally, the suite of RQA statistics available provides more objective measures of choice dynamics.

Tutorial Structure

This half-day tutorial will teach participants to use discrete RQA methods to enhance decision making and behavioral science research. We emphasize practical knowledge and hands-on experience, which will help researchers apply RQA to their own data. We will describe the value of RQA for decision making research. We will introduce the technical foundations of discrete RQA, and guide participants through exercises that develop visual expertise in interpreting auto-recurrence plots for discrete data. Participants will learn to create RPs (e.g., Figure 1), gain experience mapping RPs to task strategies, and interpret these patterns in context. We will

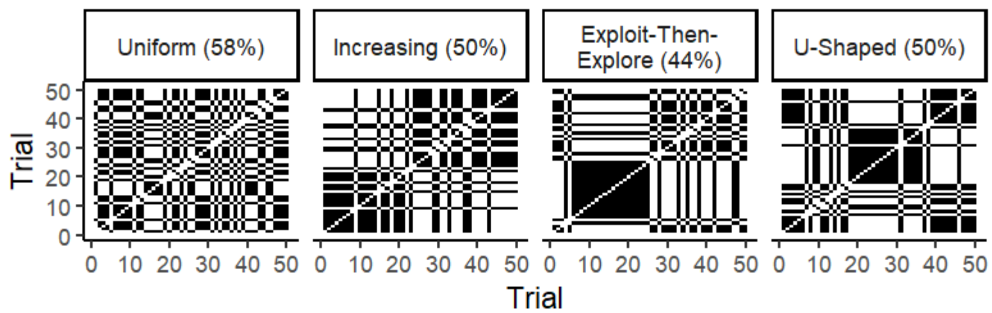

Figure 1:

Auto-recurrence plots of example choice sequences discussed in the text : *Uniform*; *Increasing*; *Exploit-Then-Explore*; *U-Shaped*. (Choice 2 proportions in parentheses.)

show how color encoding allows them to better assess specific choice patterns. Participants will use data encoding and cross-RQA to broaden their skills for comparing behavior to known strategies, such as adherence to Win-Stay-Lose-Shift or a task's optimal strategy. Finally, we will discuss extensions: comparing multiple people to each other and comparing human behavior to cognitive model predictions.

Audience

This tutorial is primarily intended for researchers working with choice data sequences, whether provided by human participants or models. This tutorial may also interest researchers working with any sequence of discrete observations and researchers interested in visual analytic techniques for time series data.

This tutorial is appropriate for scientists at any stage of their career, including undergraduate students. The emphasis is on practical applications and data interpretation. We touch on the mathematical foundations of RQA, but participants do not need deep knowledge of dynamic systems theory. Familiarity with choice data/modeling is helpful. Basic familiarity with the R statistical programming language is only needed for the section on creating RPs and computing RQA statistics.

Activities and Software

This tutorial includes short lectures, comprehension questions and discussion, and hands-on activities to practice both computing and interpreting RQA. Participants will receive the presentation slides, the full list of comprehension questions and sample answers, our R helper functions for using *crqa*, and code for all tutorial demonstrations. Prior to the tutorial sections on programming RPs and statistics, participants should have installed R, RStudio, the *crqa* (Coco & Dale, 2014) package and the *tidyverse* suite of packages (or *ggplot2*, *dplyr* and *magrittr* specifically).

Instructors

Erin McCormick is a PhD candidate at Carnegie Mellon University. She uses RQA to analyze adaptation in dynamic decision making (McCormick, Blaha, & Gonzalez, 2020a) and to assess cognitive models (McCormick et al., 2020b) She has lead multiple tutorials on using RQA in decision making research for scientists at CMU and AFRL, and has formal teaching experience as an instructor of record.

Leslie Blaha is a Senior Research Psychologist at the Air Force Research Laboratory. She leverages visual analytics to study human behaviors and cognitive models and to aid in developing interactive reasoning and decision support systems. She has used RQA to study patterns of visual analytics interactions, patterns of gaze data, and now patterns of decision making. She has formally taught statistical theory and R programming.

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