Category Theory for Cognitive Science Workshop/Tutorial

Britt Anderson (britt@uwaterloo.ca)
Department of Psychology & Centre for Theoretical Neuroscience
University of Waterloo
Waterloo, ON N2L CANADA

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Abstract

Applied Category Theory (aCT) is both a language for describing and a method for analyzing the abstract structures that populate science. It can serve as the lingua franca for cross domain discussions, and as a mathematical tool for probing the consequences of a model or theory's structure. As many cognitive scientists are unfamiliar with aCT this workshop will provide an introduction to its terminology and key features. The morning session will emphasize key concepts and tutorial exercises. The afternoon session will use recent research applications as case studies of its potential and as the basis for demonstrations of how it can be used productively.

Significance of the Topic

Many important ideas in cognitive science are about *structure*. For example, computationalism is an account of the computational and causal *relations* among mental states. Functionalism equates mental states with their *interrelations*. Psycho-physical identity theory argues for the *correspondence* of mental states and physical states.

Category Theory is the mathematics of abstract structures and their relations to concrete situations (Leinster, 2014). Originally developed in pure mathematics, it has recently received wide application (Fong & Spivak, 2018). A number of the areas of application (such as computer science (Yanofsky, 2022), philosophy (Król, 2019), physics (Coecke, 2011) , language processing (Asudeh & Giorgolo, 2020), and consciousness studies (Maruyama, 2021)) demonstrate the large potential for cognitive science.

Part of the power of category theory derives from an exquisite balance of abstraction and precision that can lead to insights not easily recognized from other theoretical points of view. For instance, category theory affords multifarious forms of compositionality beyond the classical (symbolic) versus connectionist (subsymbolic) dichotomy typically assumed by cognitive scientists. However, lack of familiarity with category theory concepts leaves many such areas of application to cognitive science largely unexplored.

This tutorial will begin to address this lack so that this powerful mathematical tool and language can bring its potential to cognitive science. No prior knowledge of category theory is presumed. Familiarity with basic mathematical concepts (e.g., sets, functions) will be helpful, but is not essential. While category theory is a deep field that takes more than one day to master, it is possible to gain a general familiarity with the basic terminology in that time frame. These basics combined with some practice applying the ideas and concepts from category theory to the sub-domains of cognitive science of interest to tutorial participants will be sufficient to allow them to have sufficient basic knowledge to start talking with disciplinary experts in category theory and to begin to develop interdisciplinary collaborations. It will also provide a foundation for tutorial participants to be able to progress further by self-study.

Tutorial Faculty

Britt Anderson, MD & PhD is co-ordinating the application. He is a neurologist, cognitive neuroscientist, and Director of the University of Waterloo (Canada) Centre for Theoretical Neuroscience (Anderson, 2021).

Steven Phillips, PhD is Chief Senior Researcher at the National Institute of Advanced Industrial Science and Technology (Japan). Dr. Phillips has a long history of applying ideas from category theory to cognitive science topics especially with the domain of cognitive development (Phillips, 2021; Phillips & Wilson, 2010)

Toby St Clere Smithe is a researcher at the Topos Institute and DPhil (PhD) candidate at the University of Oxford (UK) studying the compositional structure of computational neuroscience, with a particular focus on cognitive maps, active inference, and neural coding (T. S. C. Smithe, 2021).

Geoffrey Crutwell, PhD is an Associate Professor of Mathematics and Computer Science at Mount Allison University (Canada). His specialty area is category theory, and he has has written on how category theory can be used in machine learning (Cruttwell, Gavranović, Ghani, Wilson, & Zanasi, 2022).

Structure of the Workshop and Activities (Full Day)

Morning

Terminology and Application The basic category theory concepts will be introduced in two sessions. We start with

defining what makes something a **category** and then apply these terms in small groups to examples from the participants' areas of cognitive science research. This demonstrates the "lingua franca" character of aCT. Next, we define **functors** and **natural transformations** and participants will be guided in extending their examples. This hints at how aCT can be used to analyze model implications. Time permitting in the morning session we will present an extended detailed tutorial example that highlights some additional terminology and application.

Afternoon

After a review of the morning's material the afternoon session will feature three in-depth examples of category theory as actually applied to cognitive science. For each presentation attendees will be able to discuss with the workshop faculty how and why aCT was used. Topics planned are *systematicity and universal properties*, *machine learning and reverse derivative categories*, and *Bayesian inference and lenses*.

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